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Neutrinos in 5D SO(10) unification

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Abstract: We study neutrino physics in a 5D supersymmetric SO(10) GUT. We analyze several different choices for realizing the See-Saw mechanism. We find that the "natural" scale for the Majorana mass of right-handed neutrinos depends critically on whether the right-handed neutrinos are located in the bulk or localized on a brane. In the former case, the effective Majorana mass $M_{\rm eff}$ is "naturally" of order the compactification scale $M_c \sim 10^{14}$ GeV. Note, this is the value necessary for obtaining a light τ neutrino with $m_{\nu_\tau} \sim 10^{-2}$ eV which, within the context of hierarchical neutrino masses, is the right order of magnitude to explain atmospheric neutrino oscillations. On the other-hand when the right-handed neutrino is localized on the brane, the effective Majorana mass is typically larger than M_c. Nevertheless with small parameters of order 1/10 — 1/30, $M_{\rm eff} \sim M_c$ can be accommodated. We also discuss the constraints on model building resulting from the different scenarios for locating the right-handed neutrinos.

Keywords: GUT, Neutrino Physics, Field Theories in Higher Dimensions, Supersymmetry Phenomenology.

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Contents

1. Introduction

In a recent paper [1], we considered gauge coupling unification in a five dimensional supersymmetric SO(10) model compactified on an orbifold $S^1/(Z_2 \times Z_2')$. We obtained an excellent prediction for gauge coupling unification with a cutoff scale $M_* \sim 3 \times 10^{17}$ GeV and a compactification scale $M_c \sim 1.5 \times 10^{14}$ GeV. We also showed that our results are mathematically equivalent to a four dimensional supersymmetric grand unified theory (SUSY GUT) analysis with the color triplet Higgs mass given by M_c [1]. However, unlike the 4D case, it was shown that proton decay due to dimension 5 operators may be completely eliminated. Therefore, in this 5D framework, the unification of gauge couplings is elegantly explained. At the same time all the nice features of grand unified theories, such as charge quantization and Yukawa unification, are maintained. Unfortunately, the proton decay rate in these models, due to dimension 6 operators, is sensitive to the placement of matter multiplets in the 5th dimension, as well as to the unknown physics above the cutoff scale.

In this paper we study neutrino masses within this same 5D orbifold framework. We consider the See-Saw mechanism for neutrino masses [2] and evaluate the effective right-handed Majorana mass in different scenarios. We find that the "natural" scale for the Majorana mass of right-handed neutrinos depends critically on whether the right-handed neutrinos are located in the bulk or localized on a brane. In the former case, the effective Majorana mass $M_{\rm eff}$ is "naturally" of order the compactification scale $M_c \sim 1.5 \times 10^{14}$ GeV. Hence, the left-handed τ neutrino mass is of order $m_t(m_t)^2/3M_c \approx 0.06\,\mathrm{eV}$[1] (for $m_t(m_t) \approx 165\,\mathrm{GeV}$) which is just right (assuming hierarchical neutrino masses) to explain atmospheric neutrino oscillations and the K2K data.

On the other-hand when the right-handed neutrino is localized on the brane, the effective Majorana mass is typically larger than M_c. Nevertheless with small parameters of order 1/10 — 1/30, $M_{\rm eff} \sim M_c$ can be accommodated. We also discuss the constraints on model building resulting from the different scenarios for locating the right-handed neutrinos. Of course, in order to complete the analysis of neutrino oscillations we would need to construct a three neutrino model. Finally, we note other recent papers on 5D SO(10) SUSY GUTs within a similar orbifold framework [3]–[6]. Also an alternate mechanism for obtaining small neutrino masses in 5D GUTs is given in [7, 8].

2. See-Saw and double See-Saw mechanism in 4D

2.1 See-Saw mechanism

Quarks and charged leptons have mass ranging from $\mathcal{O}(\frac{1}{2}$ MeV to 175 GeV), which is determined theoretically in terms of a dimensionless Yukawa coupling, $\mathcal{O}(10^{-6}$ to 1), times a Higgs vev $v \approx 246\,\mathrm{GeV}$. The heaviest neutrino mass $\lesssim$ eV requires a neutrino - Higgs Yukawa coupling $\mathcal{O}(10^{-12})$ which is unnaturally small. The GRSY See-Saw mechanism, on the other hand, provides a natural explanation for such small neutrino masses with order one Yukawa couplings [2]. Since the right-handed neutrino ν_R (necessary for a Dirac neutrino mass) has zero charge under all SM gauge interactions, it can have a Majorana mass M much larger than the electroweak scale.

The simple See-Saw mass matrix for the neutrino is given by

$$M_\nu \begin{pmatrix} \nu \\ \nu^c \end{pmatrix} = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix} \tag{2.1}$$

where $\nu^c \equiv \nu_R^*$. The natural scale for M (in a 4D GUT) is of order the GUT or cutoff scale with $M \gg m_D$. Hence the two eigenvalues are very different with

$$\begin{aligned} m_1 &\simeq \frac{m_D^2}{M}\,, \\ m_2 &\simeq M\,, \\ m_1 &\ll m_D \ll m_2\,. \end{aligned} \tag{2.2}$$

[1] The factor of 3 in the denominator approximately takes into account the different RG running of the top Yukawa coupling and the effective neutrino mass operator.

Thus the largeness of M naturally explains the smallness of the left-handed neutrino mass. The problem is that in order to explain the low energy neutrino data, M must be at least two orders of magnitude smaller than the SUSY GUT scale.

2.2 Double See-Saw mechanism

In SO(10) GUTs, all quarks and leptons in one generation belong to a single spinor representation **16** ($= \mathbf{10} + \bar{\mathbf{5}} + \mathbf{1}$ of SU(5)). Unlike SU(5) where the minimal matter content includes only **10**s and $\bar{\mathbf{5}}$s with no right-handed neutrinos, SO(10) has a right-handed neutrino, neutral under the standard model gauge group, which is contained in the (minimal) **16** representation. Hence the simple See-Saw mechanism is not applicable in an SO(10) model, since the right-handed neutrino is not a singlet under SO(10); thus making it impossible to have a renormalizable Majorana mass.

For the See-Saw mechanism to work in an SO(10) model, we need to give the right-handed neutrino a Majorana mass of order the GUT scale. There are two methods for obtaining this:

- using a higher dimension operator
$$\frac{\overline{16}\ 16\ \overline{16}\ 16}{M_*}, \quad \text{or} \tag{2.3}$$
- adding an SO(10) singlet N and the renormalizable interactions
$$\lambda_N\ \overline{16}\ 16\ N + \frac{1}{2}\ M_2\ N\ N \tag{2.4}$$

where M_* is the cutoff scale of the theory. In both cases we assume a non-zero vacuum expectation value (vev)
$$\lambda_N\ \langle\overline{\mathbf{16}}\rangle = M_1 \neq 0$$
in the right-handed neutrino direction. In the first case, the product of fields has, in general, several inequivalent SO(10) invariant combinations. One particularly simple combination in the first case, with the first two fields combined to make an SO(10) singlet, can be obtained as an effective interaction after integrating out N in the second case. Note that the effective See-Saw mass M_{eff}, in either case, can be lower than the cutoff scale of the theory. In this paper we consider the latter method.

We now include the usual, electroweak scale, Dirac mass coming from the Yukawa term $\lambda\ 16_3\ 10_H\ 16_3$ (with 3 denoting the third generation). After electroweak symmetry breaking we have
$$W = m_D\ \nu\ \nu^c \tag{2.5}$$
(with $m_D = \lambda \frac{v}{\sqrt{2}} \sin\beta$) and we obtain a 3×3 neutrino mass matrix, rather than a 2×2 matrix,[2] given by
$$\mathcal{M} \begin{pmatrix} \nu \\ \nu^c \\ N \end{pmatrix} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & 0 & M_1 \\ 0 & M_1 & M_2 \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \\ N \end{pmatrix}. \tag{2.6}$$

[2] This is similar to the double see-saw mechanism suggested by Mohapatra and Valle [9].

Note, $|\mathrm{Det}\mathcal{M}| = m_D^2 M_2$, Tr $\mathcal{M} = M_2$ and $m_D \ll M_1, M_2$. Hence the effective See-Saw scale $M_{\rm eff}$ in this case may be obtained by evaluating the inverse of the heavy 2×2 mass matrix

$$\begin{pmatrix} 0 & M_1 \\ M_1 & M_2 \end{pmatrix} \tag{2.7}$$

in the ν^c direction. We find $M_{\rm eff} = M_1^2/M_2$. Note, the result is independent of the mass ordering, i.e. $M_1 \ll M_2$, $M_1 \gg M_2$ or $M_1 \approx M_2$. Finally, we obtain the light neutrino mass given by

$$m_\nu \simeq \frac{m_D^2 M_2}{M_1^2}\,, \tag{2.8}$$

irrespective of the ratio M_1/M_2, as long as $m_D \ll M_1, M_2$. The question now is how to obtain an effective See-Saw scale $M_{\rm eff} = M_1^2/M_2 \sim 10^{14}\,\mathrm{GeV}$.

3. Setup-SO(10) on $\mathcal{M}_4 \times S^1/(Z_2 \times Z_2')$

We consider a five dimensional supersymmetric SO(10) GUT compactified on an $S^1/(Z_2 \times Z_2')$ orbifold. One orbifolding, Z_2, reduces $N = 2$ supersymmetry to $N = 1$, and the other, Z_2', breaks SO(10) to the Pati-Salam (PS) gauge group $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. The space time in the 5th direction is the line segment y : $[0, \pi R/2]$ with an SO(10) symmetry in the bulk and on the brane at $y = 0$, but only $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry on the brane at $y = \pi R/2$. We call these two fixed points respectively, "SO(10)" and "Pati-Salam" branes. The brane describes a $3 + 1$ dimensional spacetime and the gauge group is the maximal gauge symmetry surviving on each brane. Further breaking to the standard model gauge group is accomplished via the Higgs mechanism on the PS brane [1].

We are interested in obtaining the Majorana mass scale for right-handed neutrino masses. The natural candidates for this scale are the cutoff scale $M_* \sim 3 \times 10^{17}\,\mathrm{GeV}$ and the compactification scale $M_c \sim 1.5 \times 10^{14}\,\mathrm{GeV}$. Let us consider just one family of quarks and leptons (including the top, bottom and tau) which we consider locating either on the PS brane or in the bulk. Quarks and leptons of one SM family are contained in two irreducible representations given by the left-handed Weyl spinors,

$$\begin{aligned} \psi &\equiv \{Q\,,\ L\} \subset (\mathbf{4}, \mathbf{2}, \mathbf{1})\,, \\ \psi^c &\equiv \left\{ \begin{pmatrix} t^c \\ b^c \end{pmatrix}, \begin{pmatrix} \nu_\tau^c \\ \tau^c \end{pmatrix} \right\} \subset\ (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})\,, \end{aligned}$$

where $Q = \binom{t}{b}$ and $L = \binom{\nu_\tau}{\tau}$ are left-handed electroweak doublets. There are also brane fields χ^c in $(\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})$ and $\bar{\chi}^c$ in $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ which break Pati-Salam down to the Standard Model gauge group by getting vacuum expectation values (vev)s along the right-handed neutrino direction. Note, we assume that the vev of χ^c and $\bar{\chi}^c$ and all the parameters appearing in the model are of order one in an appropriate unit set by the cutoff scale M_*.

With regards to gauge coupling unification, whether one has bulk matter fields and/or brane localized matter fields does not make a difference as long as one has complete SO(10) multiplets at every KK level. However, once there are additional hypermultiplets in the bulk, the 5D gauge theory rapidly approaches a strong coupling regime. In our previous work [1] it was shown that the unification of gauge couplings is only achieved with a moderate hierarchy between the unification scale M_* and the compactification scale $M_c = \frac{1}{R}$ with $M_* R \sim 10^3$. This moderately large ratio of M_*/M_c is self-consistent only if a non-trivial fixed point exists in the 5D super-Yang-Mills theory. The condition for such a strongly coupled fixed point in 5D is satisfied only when the number of SO(10) hypermultiplets in the bulk with n_{10} in vector and n_{16} in spinor representations satisfies $n_{10} \leq 6$ and $n_{16} \leq 2$ [10]. Therefore, in order to maintain self-consistency, we assume that at most one generation of matter fields, requiring two **16** hypermultiplets of SO(10), are in the bulk. This restriction does not, however, apply to SO(10) singlets. We introduce the SO(10) singlet, bulk fields N and N^c which form a hypermultiplet in 5D.

There are two choices for the orbifold parity of N and N^c.[3]

- $N_{(+,+)}$ *and* $N^c_{(-,-)}$

  N^c vanishes on both SO(10) and Pati-Salam fixed points.

- $N_{(+,-)}$ *and* $N^c_{(-,+)}$

  N vanishes on Pati-Salam fixed point and N^c vanishes on SO(10) fixed point. The Fourier expansions of N and N^c are given by

$$\begin{aligned} N &= \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} N_n \cos\left(\frac{(2n+1)y}{R}\right), \\ N^c &= \frac{1}{\sqrt{\pi R}} \sum_{n=0}^{\infty} N^c_n \sin\left(\frac{(2n+1)y}{R}\right). \end{aligned} \tag{3.1}$$

From now on, we consider the second case in which the zero modes of N and N^c are already projected out by the orbifolding.

4. Different mass terms in 5D

For ν, ν^c, N and N^c, we have various mass terms either in the bulk or on the brane. For N and N^c, we can have both bulk mass terms and brane mass terms. Furthermore, they have 5D kinetic terms with ∂_5 which behave as 4D mass terms. The fields ν and ν^c can have brane localized mass terms by themselves or with N and N^c. Since some fields reside only on the Pati-Salam brane and we have 5D $N = 1$ supersymmetry in the bulk, not all mass terms are allowed. Let us thus discuss the allowed neutrino mass terms in a 5D $N = 1$ supersymmetric theory with orbifold fixed points.

[3] We use N and N^c for both superfields and their fermionic components. Here they are superfields.

4.1 Dirac mass

- *Bulk Kaluza-Klein mass for N and N^c*

  There are Kaluza-Klein mass terms for $N,\ N^c$ which come from a 5D kinetic term,

$$\begin{aligned}\int dyW &= \int dy\ N\ \partial_5 N^c \\ &\approx \sum_{n=0}^{N_*} (2n+1)M_c\ N_n\ N_n^c\,. \end{aligned} \tag{4.1}$$

  Note, consistent with the cutoff scale M_*, we truncate KK modes at N_* satisfying $2N_*M_c \simeq M_*$.

- *Kink mass for N and N^c*

  We can write a Dirac mass term for N and N^c in the bulk,

$$W = M_{\text{kink}}\ N\ N^c\,, \tag{4.2}$$

  where M_{kink} must be odd under $Z_2 \times Z_2'$; hence a kink mass. Depending on the sign of M_{kink}, there can appear two new light modes localized at opposite fixed points. In this paper we do not consider this possibility and set $M_{\text{kink}} = 0$

- *SM Yukawa couplings*

  We can write down the usual Yukawa couplings on the Pati-Salam brane.

$$W = \lambda\ \psi\ \mathcal{H}\ \psi^c\ \delta\left(y - \frac{\pi R}{2}\right). \tag{4.3}$$

  When $\mathcal{H} \equiv (\mathbf{1}, \bar{\mathbf{2}}, \mathbf{2})$ acquires a vev $v \approx 246\,\text{GeV}$ at the electroweak scale with $F_{\mathcal{H}} = 0$, we obtain the usual Dirac mass terms between the left-handed ν and the right-handed ν^{c*},

$$W = m\ \nu\ \nu^c\ \delta(y - \frac{\pi R}{2}) \tag{4.4}$$

  with $m = \lambda \frac{v}{\sqrt{2}} \sin\beta$.

- *Pati-Salam Yukawa couplings*

  Both bulk and brane fields can have interactions on the brane. The PS gauge symmetry and the orbifold parity restrict the possible interactions.

  We consider two cases: i) ψ_3^c in the bulk and ii) ψ_3^c on the brane.

  - *ψ_3^c in the bulk*

    If ψ_3^c is in the bulk, the relevant interaction is given by

$$W = \frac{6\pi^2}{M_*}\lambda_D\ \bar{\chi}^c\ \psi^c N^c\ \delta\left(y - \frac{\pi R}{2}\right). \tag{4.5}$$

The coefficient is determined such that $\lambda_D = 1$ corresponds to the result from naive dimensional analysis with a strong coupling assumption.[4] We consider $\lambda_D = 1$ as a "natural" value of the coupling. When $\bar{\chi}^c$ develops a vev at the cutoff scale with $F_{\bar{\chi}^c} = 0$ and $\langle \bar{\chi}^c \rangle \simeq \frac{M_*}{4\pi}$, we get

$$W = \frac{3\pi}{2}\,\lambda_D\;\nu^c\;N^c\;\delta\left(y - \frac{\pi R}{2}\right). \tag{4.6}$$

We then decompose it into KK modes,

$$\begin{aligned}\int dyW &= \sqrt{\frac{3}{2 2^{\delta_{n,0}}}}\lambda_D M_c \sum_{n,m=0}^{N_*} (-1)^{n+m}\nu_n^c\;N_m^c \\ &= \sqrt{\frac{2}{2^{\delta_{n,0}}}} \sum_{n,m=0}^{N_*} (-1)^{n+m} M_D\;\nu_n^c\;N_m^c.\end{aligned}$$

Finally the Dirac neutrino mass for ν^c and N_n is given by

$$M_D = \sqrt{\frac{3}{4}}\lambda_D M_c\,, \tag{4.7}$$

For $\lambda_D = 1$, we have the "natural" value for $M_D = \sqrt{\frac{3}{4}}M_c \simeq M_c$, using, as stated above, naive dimensional analysis.

– ψ_3^c *on the PS brane*

If ψ_3^c is on the PS brane, the interaction is[5]

$$W = \sqrt{\frac{24\pi^3}{M_*}}\lambda_D\;\bar{\chi}^c\;\psi^c N^c\;\delta(y - \frac{\pi R}{2}). \tag{4.8}$$

With $\langle \bar{\chi}^c \rangle \simeq \frac{M_*}{4\pi}$, we get

$$W = \sqrt{\frac{3\pi M_*}{2}}\;\lambda_D\;\nu^c\;N^c\;\delta(y - \frac{\pi R}{2}). \tag{4.9}$$

KK decomposition gives

$$\begin{aligned}\int dyW &= \sqrt{\frac{3M_* M_c}{2}}\lambda_D \sum_{n=0}^{N_*} (-1)^n \nu^c\;N_n^c \\ &= \sum_{n=0}^{N_*} (-1)^n M_D\;\nu^c\;N_n^c\,.\end{aligned} \tag{4.10}$$

[4]You can easily obtain this result by having an overall factor of $1/(16\pi^2)$ and then multiply by 4π for each brane field and $(24\pi^3)^{1/2}$ for each bulk field in the expression [11].

[5]Note λ_D defined in eq. (4.8) is not the same quantity defined in eq. (4.5), although we use the same notation. This should not cause any confusion, since we never use both at the same time.

The Dirac mass M_D for ν^c and N_n is then given by

$$M_D = \sqrt{\frac{3M_*}{2M_c}}\lambda_D M_c\,. \tag{4.11}$$

In this case, for $\lambda_D = 1$, we have $M_D = \sqrt{\frac{3M_*}{2M_c}}M_c \sim \sqrt{\frac{M_*}{M_c}}M_c \sim 30\ M_c$ for $M_*/M_c \sim 10^3$.

Note, although the term

$$W' = C\ \lambda'_D\ \bar{\chi}^c\ \partial_5 N\ \psi^c\ \delta(y - \frac{\pi R}{2}) \tag{4.12}$$

is also possible, with $C = \frac{6\pi^2}{M_*^2}$ for ψ^c in the bulk and $C = \sqrt{\frac{24\pi^3}{M_*^3}}$ for ψ^c on the PS brane, it is subleading compared to the term without the derivative ∂_5 and is suppressed by M_c/M_*. We do not consider this term further in the paper.

4.2 Majorana mass

- *Majorana Mass in the bulk*

  A Majorana mass is allowed in the bulk for gauge singlet fields,

  $$W = \frac{1}{2}M_N(-NN + N^cN^c)\,. \tag{4.13}$$

  A 5D Majorana mass has a relative minus sign when it is expressed in terms of two 4D Weyl spinors.[6] We consider all possible values of $M_N \leq M_*$. (See the appendix for a detailed discussion of Majorana masses in 5D.)

- *Majorana Mass on the brane*

  It is possible to have a brane Majorana mass,

  $$W = \frac{1}{2}\left(a_1 NN + \frac{d_1}{M_*^2}\partial_5 N^c \partial_5 N^c\right)\delta(y) + \frac{1}{2}(a_2 N^cN^c + \frac{d_2}{M_*^2}\partial_5 N \partial_5 N)\delta\left(y - \frac{\pi R}{2}\right). \tag{4.14}$$

  However, brane Majorana mass terms are volume suppressed compared to the bulk Majorana mass by a factor M_c/M_*. In addition, terms with ∂_5 are suppressed by extra powers of M_c/M_*. Therefore, we set the brane Majorana mass terms to zero in our analysis when considering a bulk Majorana mass. However, when there is no bulk Majorana mass, the brane Majorana mass plays an important role.

Let us now summarize the mass terms in our setup. We have a bulk Dirac mass for $N_{(+,-)}$ and $N^c_{(-,+)}$ coming from the 5D kinetic term (eq. (4.1)) and also a bulk (eq. (4.13)) or brane (eq. (4.14)) Majorana mass. We also have a Dirac mass coupling N to ν^c on the PS brane. This takes different values depending on whether ψ_3^c is located in the bulk (eqs. (4.5)–(4.7)) or for ψ_3^c on the brane (eqs. (4.8)–(4.11)). As a consequence of these mass terms, we obtain an effective Majorana mass for ν^c.¡ The fields ν, ν^c then obtain a Dirac mass, at the electroweak scale via the Higgs doublet vev (eqs. (4.3)–(4.4)).

[6]It is possible to redefine $\tilde{N} = -iN$ such that the Majorana mass does not have a relative minus sign, $W = \frac{1}{2}M_N(\tilde{N}\tilde{N} + N^cN^c)$. In this case the Dirac (KK) mass term becomes $W = i\tilde{N}\partial_5 N^c$. In this paper, we use the Dirac (KK) mass term $W = N\partial_5 N^c$.

5. Neutrino mass matrix

We now calculate the eigenvalues of the neutrino mass matrix. The aim of this paper is to evaluate the left-handed tau neutrino mass using the See-Saw mechanism. We do this in two steps. Step 1: Calculate the effective Majorana mass for the right-handed tau neutrino ν_τ^c. Since the electroweak Dirac neutrino mass (m) is extremely small compared to all other scales (M_* or M_c), we first deal with the mass matrix for ν^c, N and N^c. Step 2: We then calculate the mass of ν_τ. We use techniques similar to those discussed in [12, 13]. Note, in the following the left- (right)-handed tau neutrino are simply denoted by ν (ν^c).

There are two possibilities for the Majorana mass of N and/or N^c. It can come from either a bulk Majorana mass or from a brane Majorana mass. We consider both possibilities here. We also consider the two choices of locating ν^c either in the bulk or on the PS brane. We show that the final result is independent of whether we use a bulk or brane mass for N and/or N^c. However the result depends critically on whether the field ν^c is located in the bulk or on the PS brane. In the former case we show that $M_{\rm eff} \approx M_c$ "naturally?" In the latter case, we describe how to obtain this result with minimal tuning of parameters.

5.1 Bulk Majorana mass for N and N^c

The most general superpotential has the mass terms as

$$W = N\partial_5 N^c + \frac{1}{2}M_N(-NN + N^cN^c) + C\lambda_D \nu^c N^c \delta\left(y - \frac{\pi R}{2}\right), \tag{5.1}$$

which is decomposed into KK modes as

$$W = \sum_{n=0}^{N_*}\left[(2n+1)M_c N_n N_n^c + \frac{1}{2}M_N(-N_nN_n + N_n^cN_n^c) + (-1)^n M_D \nu^c N_n^c\right]. \tag{5.2}$$

Note, the constant $C = \frac{3\pi}{2}\left(\sqrt{\frac{3\pi M_*}{2}}\right)$ for ν^c in the bulk (on the PS brane).

5.1.1 Brane localized ν^c case

The above formula applies to both the bulk ν^c and the brane localized ν^c. In the following discussion we first consider the case of a brane localized ν^c, however our discussion below applies equally well to the zero mode of a bulk field with the identification $\nu^c \equiv \nu_0^c$. Later we will add the tower of KK modes for the bulk ν^c case.

We now define

$$\mathcal{N} = (\nu^c, N_0, N_0^c, N_1, N_1^c, \ldots, N_n, N_n^c, \ldots)\,. \tag{5.3}$$

Then

$$W = \frac{1}{2}\,\mathcal{N}^T\,\hat{\mathcal{M}}\,\mathcal{N} \tag{5.4}$$

with

$$\hat{\mathcal{M}} = \begin{pmatrix} 0 & 0 & M_D & 0 & -M_D & \cdots & 0 & (-1)^n M_D & \cdots \\ 0 & -M_N & M_c & 0 & 0 & \cdots & 0 & 0 & \cdots \\ M_D & M_c & M_N & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & 0 & -M_N & 3M_c & \cdots & 0 & 0 & \cdots \\ -M_D & 0 & 0 & 3M_c & M_N & \cdots & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & -M_N & (2n+1)M_c & \cdots \\ (-1)^n M_D & 0 & 0 & 0 & 0 & \cdots & (2n+1)M_c & M_N & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} . \tag{5.5}$$

The effective right-handed neutrino mass is obtained by finding $(\hat{\mathcal{M}}^{-1})_{\nu^c\nu^c}$. Let $\mathcal{M}$ be the matrix without ν^c which has one less column and row compared to $\hat{\mathcal{M}}$.

$$(\hat{\mathcal{M}}^{-1})_{\nu^c\nu^c} = \frac{\mathrm{Det}\mathcal{M}}{\mathrm{Det}\hat{\mathcal{M}}} . \tag{5.6}$$

$$\mathrm{Det}\mathcal{M} = \prod_{n=0}^{N_*} \left(-M_N^2 - ((2n+1)M_c)^2\right) . \tag{5.7}$$

Using the formulae in the appendix we obtain

$$\begin{aligned} \mathrm{Det}\hat{\mathcal{M}} &= M_D^2 M_N \sum_{n=0}^{N_*} \frac{1}{M_N^2 + ((2n+1)M_c)^2} \mathrm{Det}\mathcal{M} \\ &\approx \frac{\pi M_D^2}{4M_c} \tanh\left(\frac{\pi M_N}{2M_c}\right) \mathrm{Det}\mathcal{M}. \end{aligned} \tag{5.8}$$

Then

$$M_{\text{eff}} = \frac{1}{(\hat{\mathcal{M}}^{-1})_{\nu^c\nu^c}} = \frac{\pi M_D^2}{4M_c} \tanh\left(\frac{\pi M_N}{2M_c}\right) \tag{5.9}$$

is the effective Majorana mass for the right-handed neutrino. Note, the limit $N_* \to \infty$ is finite and the equality is obtained in this limit.

We now consider two possible limits for the bulk Majorana mass M_N, i.e. (Case (1)) $M_c \le M_N \le M_*$ and (Case (2)) $M_N \ll M_c$. In case (1), the answer is insensitive to the size of M_N. We obtain the effective right-handed neutrino mass

$$M_{\text{eff}} = \frac{\pi M_D^2}{4M_c} . \tag{5.10}$$

In case (2), we have

$$M_{\text{eff}} = \frac{\pi^2 M_D^2 M_N}{8M_c^2} . \tag{5.11}$$

In order to further determine the size of M_{eff} we now consider the natural size of the Dirac mass M_D. It is in the calculation of M_D that the location of ν^c becomes relevant.

We first consider case (1). When ν^c is on the PS brane, the natural size of $M_D = \sqrt{\frac{3M_*}{2M_c}}\lambda_D M_c$ (eq. (4.11)) with λ_D "naturally" O(1). Hence the "natural" value for $M_{\rm eff} \sim M_*$. In order to obtain the correct size for the right-handed Majorana mass we need $\lambda_D \sim \sqrt{M_c/M_*} \sim 1/30$. This is not a particularly onerous amount of fine-tuning. However, this may be achieved "naturally" by adding a brane localized SO(10) singlet field S and replacing the Dirac mass term (eq. (4.8)) by the mass term

$$W = \sqrt{\frac{24\pi^3}{M_*}}\frac{S}{M_*}\bar{\chi}^c\psi^c N^c\,\delta\left(y - \frac{\pi R}{2}\right). \tag{5.12}$$

Note, the leading term, $W = \bar{\chi}^c\psi^c N^c$ can be forbidden by a U(1) symmetry under which S and ψ^c carry charges $1, -1$ respectively. In this case, the fundamental Yukawa coupling λ_D is replaced by the ratio $4\pi\langle S\rangle/M_*$. The suppression factor O(1/30) is now obtained "naturally" by a U(1) symmetry breaking vev for $4\pi\langle S\rangle/M_*$, i.e. one order of magnitude less than its "natural" value.

In case (2), where

$$M_{\rm eff} = \frac{\pi^2 M_D^2 M_N}{8M_c^2} = \frac{3\pi^2}{16}\lambda_D^2\frac{M_*}{M_c}M_N\,, \tag{5.13}$$

we need $\lambda_D^2\; M_N \approx \frac{M_c}{M_*}M_c \approx 10^{-3}M_c$. This can be obtained with either a small Majorana mass $M_N \approx 10^{-3}M_c$, or a small Yukawa coupling $\lambda_D \sim 1/30$ or some linear combination thereof such as $\lambda_D \sim M_N/M_c \sim 1/10$. *In any case the desired value for $M_{\rm eff} \approx M_c$ can be accommodated.*

5.1.2 Bulk ν^c case: Inclusion of ν^c KK modes

We now show that in the case of ν^c in the bulk, although the formula for $M_{\rm eff}$ (eq. (5.9)) is unchanged, the "natural" value for the Dirac mass $M_D \approx M_c$.

In the previous case we assumed ν^c is a brane field without KK modes. Now, when ν^c (i.e. ψ^c) is in the bulk, the mass matrix becomes much larger, but the analysis remains the same. The mass term is given by

$$W = \frac{1}{2}\mathcal{N}^T\hat{\mathcal{M}}\mathcal{N}\,, \tag{5.14}$$

with an extended definition of $\mathcal{N}$,

$$\mathcal{N} = (\nu_0^c, N_0, N_0^c, N_1, N_1^c, \ldots, \nu_1^c, \bar{\nu^c}_1, \nu_2^c, \bar{\nu^c}_2, \ldots)\,, \tag{5.15}$$

and the mass matrix is

$$
\hat{\mathcal{M}} = \begin{pmatrix}
0 & 0 & M_D & 0 & -M_D & \cdots & 0 & 0 & 0 & 0 & \cdots \\
0 & -M_N & M_c & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots \\
M_D & M_c & M_N & 0 & 0 & \cdots & -\sqrt{2}M_D & 0 & \sqrt{2}M_D & 0 & \cdots \\
0 & 0 & 0 & -M_N & 3M_c & \cdots & 0 & 0 & 0 & 0 & \cdots \\
-M_D & 0 & 0 & 3M_c & M_N & \cdots & \sqrt{2}M_D & 0 & -\sqrt{2}M_D & 0 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & -\sqrt{2}M_D & 0 & \sqrt{2}M_D & \cdots & 0 & 2M_c & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 2M_c & 0 & 0 & 0 & \cdots \\
0 & 0 & \sqrt{2}M_D & 0 & -\sqrt{2}M_D & \cdots & 0 & 0 & 0 & 4M_c & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 4M_c & 0 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{pmatrix} . \tag{5.16}
$$

The effective right-handed neutrino mass is obtained by finding $(\hat{\mathcal{M}}^{-1})_{\nu_n^c\nu_n^c}$ for $n = 0, 1, 2, \cdots$. We find

$$
M_{\nu_0^c\nu_0^c} = \frac{1}{(\hat{\mathcal{M}}^{-1})_{\nu_0^c\nu_0^c}} = \frac{\pi M_D^2}{4M_c} \tanh\left(\frac{\pi M_N}{2M_c}\right) , \tag{5.17}
$$

$$
(\hat{\mathcal{M}}^{-1})_{\nu_n^c\nu_n^c} = 0 \quad (n = 1, 2, \ldots) . \tag{5.18}
$$

We note two interesting facts about this result. Firstly, this result is identical to the effective right-handed neutrino Majorana mass found for the previous case with ν^c localized on the brane (eq. (5.9)). Secondly, the effective Majorana mass is generated only for the zero mode of ν^c and is independent of KK modes of ν^c. You can easily check this using the above mass matrix by adding or subtracting $\sqrt{2} \times \nu_0^c$ column (row) to the ν_n^c column (row) which makes the mass matrix block diagonal form for ν_n^c for $n \geq 1$. This operation does not change the determinant. We find

$$
\mathrm{Det}\hat{\mathcal{M}} = \mathrm{Det} \begin{pmatrix}
0 & 0 & M_D & 0 & -M_D & \cdots & 0 & 0 & 0 & 0 & \cdots \\
0 & -M_N & M_c & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots \\
M_D & M_c & M_N & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & -M_N & 3M_c & \cdots & 0 & 0 & 0 & 0 & \cdots \\
-M_D & 0 & 0 & 3M_c & M_N & \cdots & 0 & 0 & 0 & 0 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 2M_c & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 2M_c & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 4M_c & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 4M_c & 0 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{pmatrix} . \tag{5.19}
$$

We then easily find the $\nu_n^c\nu_n^c$ component of the inverse which is just the inverse of each 2×2 matrix and it is zero.

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For the most natural values of M_D ($= \sqrt{\frac{3}{4}}M_c$) (eq. (4.7)) and for any value of M_N satisfying $M_* \geq M_N \geq M_c$, we get the effective right-handed Majorana mass

$$M_{\text{eff}} = \frac{3\pi}{16} M_c \,. \tag{5.20}$$

Therefore in this case, the effective Majorana mass for the right-handed neutrino is "naturally" given by the compactification scale $M_c \sim 10^{14}\,\text{GeV}$.

5.2 Brane Majorana mass for N

It is possible to imagine $M_N = 0$. Suppose there is an additional U(1) symmetry (lepton number) under which $N(N^c), \bar{\chi}^c$ carries a charge $1(-1), 3$. The bulk Majorana mass is forbidden. Then if there is a brane localized field S, carrying U(1) charge -2, we can write down the Majorana mass term on the brane.[7] We consider the case of both a brane localized and bulk field $\psi^c \supset \nu^c$. As before *we first consider* ν^c *localized on the brane.*

The most general superpotential has the mass terms given by

$$W = N\ \partial_5 N^c + \frac{3\pi^2}{M_*}\lambda_N\ S\ N\ N\ \delta(y) + 4\pi\sqrt{\frac{24\pi^3}{M_*^3}}\lambda_D\ S\ \bar{\chi}^c\ \psi^c\ N^c\ \delta\left(y - \frac{\pi R}{2}\right), \tag{5.21}$$

which is decomposed into KK modes as

$$W = \sum_{n=0}^{\infty}\left[(2n+1)M_c N_n N_n^c + \sum_{m=0}^{\infty}(M_N N_n N_m) + (-1)^n M_D \nu^c N_n^c\right] \tag{5.22}$$

with

$$M_N = \frac{3\pi\langle S\rangle}{M_*}\lambda_N\ M_c \tag{5.23}$$

and

$$M_D = \frac{\sqrt{24\pi^2}\langle S\rangle}{M_*}\frac{4\pi\langle\bar{\chi}^c\rangle}{M_*}\sqrt{\frac{M_*}{M_c}}\lambda_D M_c \approx \frac{\sqrt{24\pi^2}\langle S\rangle}{M_*}\sqrt{\frac{M_*}{M_c}}M_c \tag{5.24}$$

where the last term is the "natural" value (see for comparison eq. (4.11)).

We now define

$$\mathcal{N} = (\nu^c, N_0, N_0^c, N_1, N_1^c, \ldots, N_n, N_n^c, \ldots)\,. \tag{5.25}$$

Then

$$W = \frac{1}{2}\ \mathcal{N}^T\ \hat{\mathcal{M}}\ \mathcal{N} \tag{5.26}$$

[7] It is not clear whether it would be better to have S in the bulk or on the brane. For definiteness here we consider S localized on the brane.

with

$$\hat{\mathcal{M}} = \begin{pmatrix} 0 & 0 & M_D & 0 & -M_D & \cdots & (-1)^n M_D & 0 & \cdots \\ 0 & 2M_N & M_c & 2M_N & 0 & \cdots & 2M_N & 0 & \cdots \\ M_D & M_c & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 2M_N & 0 & 2M_N & 3M_c & \cdots & 2M_N & 0 & \cdots \\ -M_D & 0 & 0 & 3M_c & 0 & \cdots & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ (-1)^n M_D & 2M_N & 0 & 2M_N & 0 & \cdots & 2M_N & (2n+1)M_c & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & (2n+1)M_c & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix}. \tag{5.27}$$

The effective right-handed neutrino mass is obtained similarly by finding $(\hat{\mathcal{M}}^{-1})_{\nu^c\nu^c}$ where

$$(\hat{\mathcal{M}}^{-1})_{\nu^c\nu^c} = \frac{\mathrm{Det}\mathcal{M}}{\mathrm{Det}\hat{\mathcal{M}}}. \tag{5.28}$$

We obtain

$$\mathrm{Det}\mathcal{M} = \prod_{n=0}^{\infty}(-((2n+1)M_c)^2) \tag{5.29}$$

and

$$\begin{aligned} \mathrm{Det}\hat{\mathcal{M}} &= -2M_N M_D^2 \left[\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)M_c}\right]^2 \mathrm{Det}\mathcal{M} \\ &= -\frac{\pi^2 M_N M_D^2}{8M_c^2}\mathrm{Det}\mathcal{M}. \end{aligned} \tag{5.30}$$

Therefore we obtain the effective Majorana mass for the right-handed neutrino given by the expression

$$M_{\mathrm{eff}} = \frac{1}{|(\hat{\mathcal{M}}^{-1})_{\nu^c\nu^c}|} = \frac{\pi^2 M_N M_D^2}{8M_c^2}. \tag{5.31}$$

Note, this result is the same as in the case of the bulk Majorana mass, when $M_N \ll M_c$ (see eq. (5.11)). Finally, we have $M_N \sim \frac{3\pi\langle S\rangle}{M_*} M_c$ (Eqn (5.23)). Thus M_N is "naturally" $\mathcal{O}(\frac{3}{4}M_c)$ for $\frac{4\pi\langle S\rangle}{M_*} \sim 1$.

We now obtain the following results for the effective right-handed neutrino mass. When *ν^c is localized on the brane*, we have $M_D \sim \frac{\sqrt{24\pi^2\langle S\rangle}}{M_*}\sqrt{\frac{M_*}{M_c}}M_c$ (eq. (5.24)). Hence we can get $M_{\mathrm{eff}} \sim M_c$, but only if we take $4\pi\langle S\rangle/M_* \sim 1/10$. However if we now take ν^c *in the bulk*, using the "natural" values for $M_D \sim \sqrt{\frac{3}{4}}M_c$ (see eq. (4.7)) and $M_N \sim \frac{3}{4}M_c$, we get $M_{\mathrm{eff}} \sim \frac{9\pi^2}{128}M_c$. In both cases, we find results very similar to the case of bulk Majorana mass. In summary, *for ν^c localized on the brane an effective right-handed neutrino mass of order M_c can be accommodated, while for ν^c in the bulk we "naturally" obtain $M_{\mathrm{eff}} \sim M_c$.*

5.3 Left-Handed tau neutrino mass

Let us now complete the picture. As discussed above, the effective Majorana mass for ν^c (or its zero mode ν_0^c if it is a bulk field) is given by $M_{\mathrm{eff}} \equiv \gamma M_c$ where the parameter $\gamma = \mathcal{O}(1)$ depends on the details of the model. Note, only the zero mode of ν^c is effective in

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the See-Saw mechanism. We now construct the effective 2×2 mass matrix for $(\nu,\ \nu_0^c)$. We have

$$\mathcal{M}_{22}^{\rm eff} = \begin{pmatrix} 0 & m \\ m & \gamma M_c \end{pmatrix} \tag{5.32}$$

with $m = m_t$ at the GUT scale.

We now consider the most general possibilities for having $\psi^c(\supset \nu^c)$ in the bulk or on the PS brane. Similarly, $\psi(\supset \nu)$ can be either in the bulk or on the PS brane. Note, gauge coupling unification can be affected by the splitting of the left-handed ψ and the right-handed ψ^c matter fields, if all KK modes are not in complete SO(10) multiplets.

There are three possibilities for the third generation matter fields configurations. In all cases, the condition for the nontrivial fixed points is satisfied.

- ψ and ψ^c in the bulk:

  Gauge coupling unification works fine and the See-Saw scale is naturally of order M_c. However, there is a serious problem. The natural value for the top quark Yukawa coupling has a volume suppression with $\lambda = \lambda_t \sim \frac{(24\pi^3)^{3/2}}{16\pi^2}(\frac{M_c}{M_*})^{3/2}$. Hence obtaining an order one coupling is difficult. At the very least, we would need to localize the Higgs towards the PS brane. However this would destroy the success of gauge coupling unification. As a result we are forced to place the third generation on the PS brane.[8] The first two families however can be bulk fields with a welcome suppression of their Higgs Yukawa coupling.

- ψ^c in the bulk and ψ on the PS brane:

  Again, the See-Saw scale is naturally of order M_c, however now gauge couplings get a threshold correction which makes the prediction worse. The anomaly generated by massive KK modes of ψ^c is cancelled by a 5D Chern-Simons term [14].

- ψ in the bulk and ψ^c on the PS brane:

  The gauge coupling unification goes in the right direction. We get a precise unification with a smaller ratio of M_*/M_c. The effective right-handed neutrino mass needs an additional suppression by $4\pi\langle S\rangle/M_*$ of order 1/10 to give a See-Saw scale at M_c. Again there is an anomaly inflow with a 5D Chern-Simons term.

[8]Note, we could ameliorate the large suppression factor $(\frac{M_c}{M_*})^{\frac{3}{2}}$ by decreasing the ratio M_*/M_c. In our previous paper [1] we showed that this ratio is fixed by the GUT threshold correction $\epsilon_3 \equiv (\alpha_3(M_{\rm GUT}) - \tilde{\alpha}_{\rm GUT})/\tilde{\alpha}_{\rm GUT} \approx -0.04$ required to fit the precision electroweak data in the equivalent 4D theory (where $\alpha_1(M_{\rm GUT}) = \alpha_2(M_{\rm GUT}) = \tilde{\alpha}_{\rm GUT}$). We also note that the precise value of ϵ_3 depends on the squark, slepton and gaugino spectrum through electroweak threshold corrections. It has recently been noted by Dermisek [15] that as the universal GUT value of the squark and slepton mass - m_{16} - increases, with $m_{16} \gg M_{1/2}$, the value of ϵ_3 approaches zero. Moreover using the results of ref. [1] we see that $M_*/M_c \approx \exp(7|\epsilon_3|/0.04)$ with $M_c \approx M_{\rm GUT}\exp(-5|\epsilon_3|/0.04)$. Hence as $|\epsilon_3|$ decreases the compactification scale increases and the ratio M_*/M_c decreases. For example, with $\epsilon_3 = -0.02$ we have $M_*/M_c \approx 33$ and $M_c \approx 2.5 \times 10^{15}$ GeV. In this case we could put the third generation in the bulk, without suffering any volume suppression in the Yukawa coupling, with the See-Saw scale determined naturally by M_c which is moderately larger than before. Thus whether the third generation should be a bulk field or a brane field is somewhat model dependent.

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- ψ and ψ^c on the PS brane:

  Gauge coupling unification works fine. However, as above, the See-Saw scale needs a suppression from $4\pi\langle S\rangle/M_*$ of order 1/10.

With the above caveats it is possible to obtain a light tau neutrino mass $m_{\nu_\tau} \sim m^2/M_c$. Each one of the above choices, however, makes specific constraints on 5D SO(10) model building.

6. Conclusion

There are now several physics issues related to the compactification scale $M_c \approx 10^{14}$ GeV.

- Gauge coupling unification

  In 4D, SUSY GUTs require a threshold correction $\epsilon \equiv (\frac{\alpha_3(M_{\rm GUT})-\tilde{\alpha}_{\rm GUT}}{\tilde{\alpha}_{\rm GUT}}) \simeq -0.04$ in order to achieve a perfect gauge coupling unification. In minimal SU(5), this results from a color triplet Higgs mass $m_{H_3} \sim 10^{14}$ GeV. The same result would be necessary in any SUSY GUT with an assumption that there is no other threshold correction from the GUT breaking sector.

  In 5D, Orbifold SUSY GUTs determine the threshold correction as a function of M_c and M_*. There are no other free parameters. Now the threshold correction requires the Higgs to be in the bulk and determines the compactification scale $M_c \sim 10^{14}$ GeV. In this framework, the 4D GUT scale is fictitious, however it is related to the compactification and cutoff scales by the approximate expression $M_{\rm GUT} \approx (\frac{M_*}{M_c})^{2/3} M_c$. This relation determines the cutoff scale $M_* \sim 10^{17}$ GeV.

- Neutrino mass

  In 4D, the natural size of the right-handed tau neutrino Majorana mass is determined from the higher dimensional operator $\frac{1}{M_{Pl}} \overline{\mathbf{16}}\,\mathbf{16}\,\overline{\mathbf{16}}\,\mathbf{16}$ (or similarly $\frac{1}{M_{Pl}} \bar{\chi}^{\mathbf{c}}\,\psi^{\mathbf{c}}\,\bar{\chi}^{\mathbf{c}}\,\psi^{\mathbf{c}}$ in PS). By replacing $\langle\overline{16}\rangle \sim M_{\rm GUT}$ one obtains $M_{\nu^c} \sim \frac{M_{\rm GUT}^2}{M_{Pl}} \sim 10^{14}$ GeV.

  In 5D, if ν^c is localized on the PS brane, we can accommodate an effective right-handed neutrino mass of order M_c either using a U(1) symmetry with a symmetry breaking vev $\frac{4\pi\langle S\rangle}{M_*} \sim 1/10$ to suppress the natural scale for the heavy Dirac neutrino mass[9] or with a small Majorana neutrino mass with $M_N/M_c \ll 1$ (or a bit of both).

  On the other hand, for ν^c in the bulk, the volume suppression naturally gives $M_{\rm eff} \sim M_c \sim 10^{14}$ GeV even for $\langle\bar{\chi}^c\rangle \sim M_*$. Unfortunately, the same volume suppression would naturally give a top Yukawa coupling $\lambda_t \sim \frac{(24\pi^3)^{3/2}}{16\pi^2}(\frac{M_c}{M_*})^{3/2} \ll 1$. This is a serious problem which can be ameliorated by decreasing the volume factor $(\frac{M_*}{M_c})^{3/2}$. One particular solution is testable at the electroweak scale. As discussed in Footnote 8, changing the universal squark and slepton mass m_{16} directly affects gauge coupling threshold corrections at the electroweak scale and indirectly at the GUT

[9] This is the Froggatt-Nielsen mechanism [16] applied to neutrino masses.

scale. Moreover, increasing m_{16} has the effect of increasing the compactification scale M_c, decreasing the cutoff scale M_*, hence decreasing the ratio $(M_*/M_c)^{3/2}$. It is encouraging that the scale necessary for the atmospheric neutrino oscillations (which is the tau neutrino mass in SO(10)) can be the same as the compactification scale $M_c \sim 10^{14-15}$ GeV, necessary for a successful unification of gauge couplings.

Finally, it is interesting to note that part of this result could simply have been obtained using an effective higher dimensional right-handed neutrino mass operator on the PS brane given by

$$W = \frac{C}{2M_*^n} \bar{\chi}^{\mathbf{c}}\, \psi^{\mathbf{c}}\, \bar{\chi}^{\mathbf{c}}\, \psi^{\mathbf{c}} \delta\left(y - \frac{\pi R}{2}\right) \tag{6.1}$$

with $n = 1$, $C = 16\pi^2 c$ (for ψ^c on the PS brane) or $n = 2$, $C = 24\pi^3 c$ (for ψ^c in the bulk). Thus when $4\pi\langle\bar{\chi}^c\rangle \sim M_*$ in the right-handed neutrino direction and $c \sim 1$ (i.e. the "natural" values) we find a right-handed neutrino mass

$$W = \frac{1}{2} M_{\mathrm{eff}}\; \nu^c\; \nu^c \tag{6.2}$$

with

$$M_{\mathrm{eff}} \sim \begin{pmatrix} M_* \\ \frac{3}{4} M_c \end{pmatrix} \quad \text{for} \quad \psi^c \begin{pmatrix} \text{on the PS brane} \\ \text{in the bulk} \end{pmatrix}. \tag{6.3}$$

Once again we find that the value of M_{eff} for ψ^c in the bulk is of order M_c, but for ψ^c on the PS brane we can only obtain this desired value with a small value of $c \sim 10^{-3}$. However note, with this effective operator analysis we cannot realize the possibility of suppressing M_{eff} (for ψ^c on the PS brane) with a small Majorana mass.

- 5D Planck scale

  There is another very interesting relation between the 5D Planck scale $M_{5\mathrm{D}}$ and the cutoff scale M_*. In the presence of the extra dimension, the 4D Planck scale is a derived scale which is determined by the 5D Planck scale $M_{5\mathrm{D}}$ and the compactification scale M_c. The relation is $M_{\mathrm{Pl}}^2 = \frac{M_{5\mathrm{D}}^3}{M_c}$. Now for $M_c \sim 10^{14}$ GeV, we get $M_{5\mathrm{D}} \sim 10^{17}$ GeV. Note, this is the same as the cutoff scale M_* determined assuming gauge coupling unification.

In conclusion, in this paper we have discussed several different possible frameworks for obtaining a See-Saw mechanism in 5D SUSY SO(10). If the right-handed tau neutrino is in the bulk then we find that the "natural" value of the See-Saw scale is given by the compactification scale $M_c \sim 10^{14}$ GeV. As noted in the text, this scenario has a serious problem with a very small top quark Yukawa coupling. This problem can be overcome, however, with the bottom line effect of moderately increasing M_c to about 10^{15} GeV. On the other hand, if the right-handed tau neutrino is on the Pati-Salam brane, then the "natural" value for the See-Saw scale is greater than the compactification scale. However, using a Froggatt-Nielsen like mechanism and/or a small Majorana neutrino mass $M_N \ll 1$, a See-Saw scale of order M_c can be obtained.

Note, the values of the compactification scale M_c and the cutoff scale M_* are determined by gauge coupling unification [1]. In addition, we note that the cutoff scale is the same as the 5D Planck scale determined using the observed the 4D Planck scale and the compactification scale. This triple significance for a scale around 10^{14} GeV is either an amazing coincidence or very profound.

Finally, we have only discussed the effective right-handed Majorana mass appropriate for one family of quarks and leptons in this paper. Further discussion of neutrino mixing angles within this framework must wait until a three family model is constructed.

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A. 5D fermion mass operator

A.1 5D mass in terms of 4D Weyl spinors

To express 5D mass terms in terms of 4D Weyl spinor, we summarize our spinor convention and charge conjugation (especially for the 5D Majorana mass).

Dirac γ matrix in 5D is

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma_i = \begin{pmatrix} 0 & -\sigma_i \\ \sigma_i & 0 \end{pmatrix}, \qquad \hat{\gamma_5} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \tag{A.1}$$

where σ_i with $i = 1, 2, 3$ is the 2×2 Pauli matrix. Each element is thus a 2×2 matrix. Note that $\hat{\gamma_5} = i\gamma_5$ for the usual γ_5 defined in 4D theory. Now $\gamma_M = \{\gamma_\mu, \hat{\gamma_5}\}$ with $M = 0, 1, 2, 3, 5$ forms a Clifford algebra in 5D

$$\{\gamma_M, \gamma_N\} = 2g_{MN} \,. \tag{A.2}$$

Generators of Lorentz transformations for 5D spinors are given by

$$\Sigma_{MN} \propto i[\gamma_M, \gamma_N]. \tag{A.3}$$

A 5D spinor and its complex conjugate is given by

$$\psi = \begin{pmatrix} \chi_1 \\ i\sigma_2\chi_2^* \end{pmatrix}, \qquad \psi^* = \begin{pmatrix} \chi_1^* \\ i\sigma_2\chi_2 \end{pmatrix}. \tag{A.4}$$

Let us consider charge conjugation. The Dirac γ matrices have the property that $\gamma_M^* = \gamma_M$ for $M = 0, 1, 3$ and $\gamma_M^* = -\gamma_M$ for $M = 2, 5$. For $C = \gamma_2\hat{\gamma_5}$, we have

$$C\gamma_M^* C^{-1} = \gamma_M \,, \tag{A.5}$$

with $C^{-1} = -C$.

Charge conjugation is defined as

$$\psi_c \equiv C\psi^* = \begin{pmatrix} \chi_2 \\ -i\sigma_2\chi_1^* \end{pmatrix}. \tag{A.6}$$

It is easy to check that ψ_c transforms as the same as ψ under the 5D Lorentz transformation.

Using the property $\gamma_0\gamma_M^\dagger\gamma_0 = \gamma_M$, we get

$$\begin{aligned}
\bar\psi\psi &= \left(\chi_1^\dagger \; -\chi_2^T i\sigma_2\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \chi_1 \\ i\sigma_2\chi_2^* \end{pmatrix} \\
&= \chi_2^T(-i\sigma_2)\chi_1 + \text{h.c.} \to \chi_2\chi_1 + \text{h.c.}\,, \qquad (A.7) \\
\bar\psi\psi_c &= \left(\chi_1^\dagger \; -\chi_2^T i\sigma_2\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \chi_2 \\ -i\sigma_2\chi_1^* \end{pmatrix} \\
&= \chi_2^T(-i\sigma_2)\chi_2 + \chi_1^\dagger(-i\sigma_2)\chi_1^* \to \chi_2\chi_2 - \chi_1\chi_1 + \text{h.c.}\,. \qquad (A.8)
\end{aligned}$$

We use $\chi_1 \to N$ and $\chi_2 \to N^c$. The corresponding Dirac mass in a supersymmetric theory is given by

$$W = M_D N N^c\,, \tag{A.9}$$

and the Majorana mass is

$$W = \frac{1}{2}M_N(-NN + N^cN^c)\,. \tag{A.10}$$

A.2 Equivalence of 5D Dirac Mass and Majorana Mass

Since a 5D Majorana mass term requires two 4D Weyl spinors, it is natural to ask whether there is a transformation which changes a 5D Dirac Mass to a 5D Majorana Mass, and vice versa.

Let us start from the theory with one hypermultiplet composed of two 4D chiral multiplets N_+ and N_- with a 5D Dirac mass. Neglecting the boundaries, in 5D, we have

$$W = N_+\partial_5 N_- + mN_+N_-\,, \tag{A.11}$$

which is the same as

$$W = \frac{1}{2}(N_+\partial_5 N_- - N_-\partial_5 N_+) + mN_+N_-\,, \tag{A.12}$$

up to a total derivative. We can re-define the fields,

$$\begin{aligned}
N &= \frac{1}{\sqrt{2}}(N_+ - N_-)\,, \\
N^c &= \frac{1}{\sqrt{2}}(N_+ + N_-)\,. \qquad (A.13)
\end{aligned}$$

The 5D superpotential is then expressed in terms of N and N^c by

$$\begin{aligned}
W &= W_{\text{bulk}} + W_{\text{boundary}} \\
W_{\text{bulk}} &= \frac{1}{2}(N\partial_5 N^c - N^c\partial_5 N) + \frac{1}{2}m(-NN + N^cN^c) \\
W_{\text{boundary}} &= \frac{1}{4}\partial_5(-NN + N^cN^c)\,. \qquad (A.14)
\end{aligned}$$

Hence, we obtain the Majorana mass from the Dirac mass by the field re-definitions, except for the last term which is a total derivative in 5D. Therefore, it is shown that the Dirac mass and the Majorana mass are equivalent in 5D up to boundary terms. Finally, in the presence of the boundaries, specifically for the orbifold compactification, the Majorana mass term can be re-expressed in terms of the Dirac mass term plus the residual effects given by

$$\begin{aligned} W_{\text{boundary}} &= \frac{1}{4}(-NN + N^cN^c)\left[-\delta(y) + \delta\left(y - \frac{\pi R}{2}\right)\right] \\ &= \frac{1}{4}\left[NN\delta(y) + N^cN^c\delta\left(y - \frac{\pi R}{2}\right)\right], \end{aligned} \tag{A.15}$$

for N_{+-} and N^c_{-+}.

A.3 Useful formulae

$$\sum_{n=-\infty}^{\infty} \frac{1}{x+n} = \pi \cot(\pi x)\,. \tag{A.16}$$

$$\sum_{n=1}^{\infty} \left[\frac{1}{x+n} + \frac{1}{x-n}\right] = -\frac{1}{x} + \pi \cot(\pi x)\,. \tag{A.17}$$

$$\sum_{n=0}^{\infty} \frac{x}{x^2 + (2n+1)^2} = \frac{\pi}{4} \tanh\left(\frac{\pi x}{2}\right). \tag{A.18}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \frac{\pi}{4}\,. \tag{A.19}$$

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